$\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing beyond the Standard Model

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1. B-physics in the LHC era

Strategies to explore the TeV scale:

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High energy:
direct production of new particles
LHC

High precision:
quantum effects from new particles
high statistics

With precision measurements one studies the couplings and mixing patterns of the new particles which the LHC will discover.

If new physics is associated with the scale Λ, effects on weak processes are generically suppressed by a factor of order M_W^2/Λ^2 compared to the Standard Model.

$\Rightarrow$ study processes which are suppressed in the Standard Model.

In the flavor-changing neutral current (FCNC) processes of the Standard Model several suppression factors pile up:

- FCNCs proceed through electroweak loops, no FCNC tree graphs,
- small CKM elements, e.g. $|V_{ts}| = 0.04$, $|V_{td}| = 0.01$,
- GIM suppression in loops with charm or down-type quarks, $\propto m_c^2/M_W^2$, m_s^2/M_W^2.
- helicity suppression in radiative and leptonic decays, because FCNCs involve only left-handed fields, so helicity flips bring a factor of m_b/M_W or m_s/M_W.

The suppression of FCNC processes in the Standard Model is not a consequence of the $SU(3) \times SU(2)_L \times U(1)_Y$ symmetry. It results from the particle content of the Standard Model and the accidental smallness of most Yukawa couplings. It is absent in generic extensions of the Standard Model.

Examples:

extra Higgses	$\Rightarrow$	Higgs-mediated FCNC's at tree-level ,
		helicity suppression possibly absent,
squarks/gluinos	$\Rightarrow$	FCNC quark-squark-gluino coupling,
		no CKM/GIM suppression,
vector-like quarks	$\Rightarrow$	FCNC couplings of an extra Z',
$\mathrm{SU(2)_R}$ gauge bosons	$\Rightarrow$	helicity suppression absent

$B_d-\overline{B}_d$ mixing and $B_s-\overline{B}_s$ mixing are sensitive to scales up to $\Lambda \sim 100\,\mathrm{TeV}$.

2. $B_s - \overline{B}_s$ mixing basics

Schrödinger equation:

$$i \frac{d}{dt} \begin{pmatrix} | B_s(t) \rangle \\ | \overline{B}_s(t) \rangle \end{pmatrix} = \left(M - i \frac{\Gamma}{2} \right) \begin{pmatrix} | B_s(t) \rangle \\ | \overline{B}_s(t) \rangle \end{pmatrix}$$

where $B_s \sim \overline{b} s$ and $\overline{B}_s \sim b \overline{s}$.

3 physical quantities in $B_s - \overline{B}_s$ mixing:

$$|M_{12}|, \quad |\Gamma_{12}|, \quad \phi = \arg \left(-\frac{M_{12}}{\Gamma_{12}} \right)$$

Two mass eigenstates:

Lighter eigenstate: $| B_L \rangle = p | B_s \rangle + q | \overline{B}_s \rangle$.

Heavier eigenstate: $| B_H \rangle = p | B_s \rangle - q | \overline{B}_s \rangle$ with $|p|^2 + |q|^2 = 1$.

with masses $M_{L,H}$ and widths $\Gamma_{L,H}$.

To determine $|M_{12}|$, $|\Gamma_{12}|$ and ϕ measure

$$\Delta m = M_H - M_L \simeq 2|M_{12}|, \qquad \Delta\Gamma = \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}|\cos\phi$$

and

$$a_{\rm fs} = \operatorname{Im}\frac{\Gamma_{12}}{M_{12}} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi.$$

$a_{\rm fs}$ is the CP asymmetry in decays $B \to f$ which are flavor-specific, i.e.

$$\overline{B} \not\to f \text{ and } B \not\to \overline{f}.$$

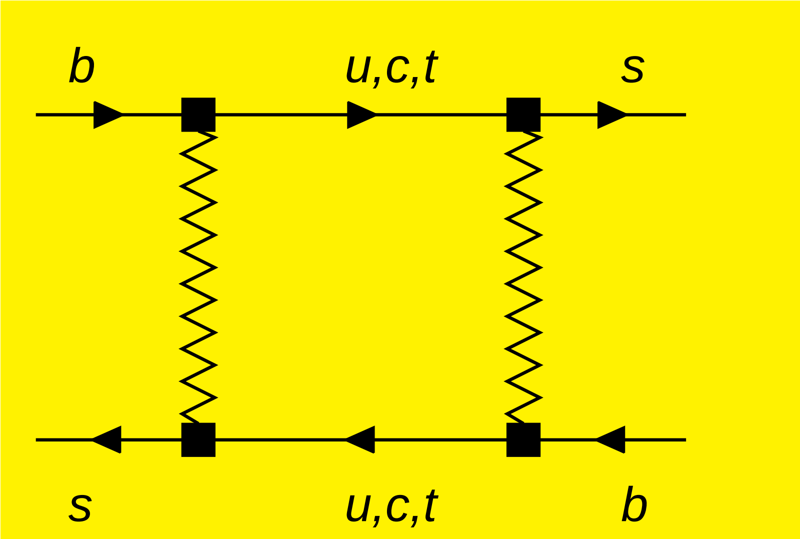
Examples: $B_s \to X\ell^+\nu_\ell$ or $B_s \to D_s^-\pi^+$.

$$a_{\rm fs} = \frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to \overline{f})}$$

The time dependence of the decay rates $\Gamma(\overline{B}(t) \to f)$ and $\Gamma(B(t) \to \overline{f})$ drops out.

$a_{\rm fs}$ measures CP violation in mixing.

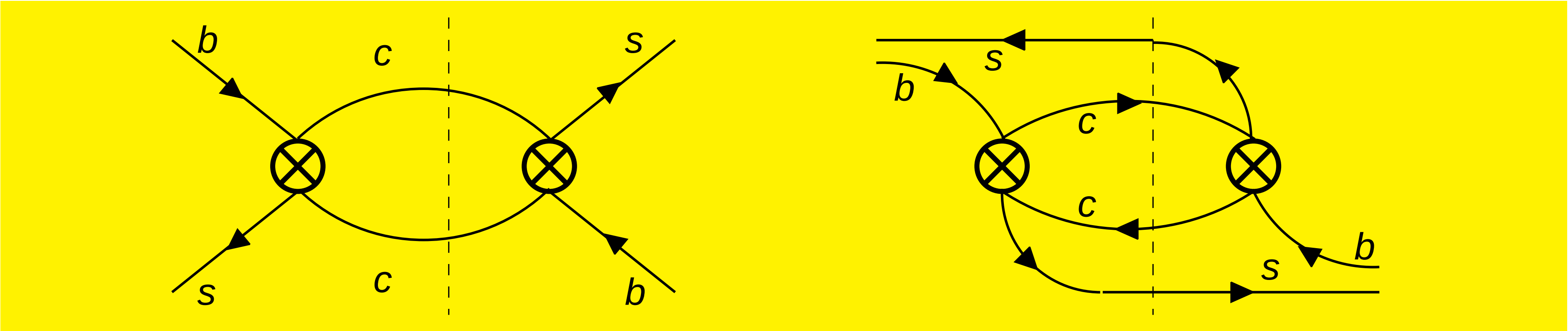
M_{12} stems from the dispersive (real) part of the box diagram, internal $(\bar{t}, t)$.
(u's and c's are negligible).



Optical theorem:

$$\Gamma_{12} = -\frac{1}{2M_{B_s}} \text{Abs} \langle B_s | \, i \int d^4x \; T \, \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | \bar{B}_s \rangle$$

from final states common to B_s and $\overline{B}_s$.



Crosses: Effective $|\Delta B| = 1$ operators from W-mediated b-decay.

Γ_{12} is a CKM-favored tree-level effect associated with final states containing a $(\overline{c}, c)$ pair.

Theory prediction

$$\Delta\Gamma \;=\; \Gamma_L - \Gamma_H \;\simeq\; 2|\Gamma_{12}|\cos\phi$$

with $\cos\phi \simeq 1$ in the Standard Model.

Corrections to M_{12} of order $\alpha_s(m_b)$: Buras, Jamin, Weisz 1990

Corrections to Γ_{12} of order Λ_{QCD}/m_b: Beneke, Buchalla, Dunietz 1996

Corrections to Γ_{12} of order $\alpha_s(m_b)$: Beneke, Buchalla, Greub, Lenz, U.N. 1998
Ciuchini,Franco,Lubicz,Mescia,Tarantino 2003

Corrections to $a_{\rm fs}$ of order $\alpha_s(m_b)$ and Λ_{QCD}/m_b:
Ciuchini, Franco, Lubicz, Mescia, Tarantino 2003
Beneke, Buchalla, Lenz, UN 2003

SM prediction for Δm_{B_s}:

$$\Delta m_{B_s} = \left(\frac{f_{B_s}}{246\,\mathrm{MeV}} \right)^2 \left(\frac{|V_{ts}|}{0.041} \right)^2 \frac{\hat{B}_{B_s}}{1.29} \times 21\,\mathrm{ps}^{-1}$$

Use lattice results for hadronic parameters:

$$\begin{aligned} B &= 0.85 \pm 0.06, & n_f &= 0 \\ \Rightarrow \quad \hat{B} &= 1.52\,B \;=\; 1.29 \pm 0.09, & & \text{JLQCD 2003} \end{aligned}$$

With $|V_{ts}| = 0.041 \pm 0.002$

$$\Rightarrow \quad \Delta m_{B_s} = \left(\frac{f_{B_s}}{246\,\mathrm{MeV}} \right)^2 (21 \pm 2)\,\mathrm{ps}^{-1}.$$

Use lattice results for f_{B_s} (Lattice 2004 average):

$$f_{B_s} = 246 \pm 16\,\text{MeV}, \qquad n_f = 2 \text{ and } n_f = 2+1$$
$$\Rightarrow \quad \Delta m_{B_s} = (21 \pm 2)\,\text{ps}^{-1}.$$

With a recent MILC result (hep-ph/0311130):

$$f_{B_s} = 260 \pm 29\,\text{MeV}, \qquad n_f = 2+1$$
$$\Rightarrow \quad \Delta m_{B_s} = (23 \pm 3)\,\text{ps}^{-1}.$$

Prediction for $\Delta\Gamma_{B_s}$:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{210\,\mathrm{MeV}}\right)^2 \left[0.006\,B \;+\; 0.172\,B_S \;-\; 0.063\right]$$
$$= 0.12^{+0.04}_{-0.03}$$

Use:

$$B_S = 0.86 \pm 0.07\,\mathrm{MeV}, \qquad n_f = 0, \quad \text{Lattice 2004}$$

But with the MILC result $f_{B_s} = 260 \pm 29\,\mathrm{MeV}$:

$$\Rightarrow \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 0.14 \pm 0.05$$

Prediction for $a_{\rm fs}$ in the B_s system:

$$a_{\rm fs} = \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin\phi \; = \; 2 \cdot 10^{-5}$$

because

$$\phi = \mathcal{O}\left(|V_{us}|^2 \frac{m_c^2}{m_b^2} \right) \; = \; 3 \cdot 10^{-3} = 0.2^\circ$$

New physics can affect M_{12} lifting the GIM suppression of ϕ.

$\Rightarrow$ large enhancement of $a_{\rm fs}$ possible

$\Rightarrow$ Even crude upper bounds constrain models of new physics.

3. $B_s - \overline{B}_s$ mixing beyond the Standard Model

M_{12} is very sensitive to new physics. In the generic Minimal Supersymmetric Standard Model (MSSM) box diagrams with gluinos and squarks can compete with the SM contribution.

$\Rightarrow$ Both $|M_{12}|$ and $\arg M_{12}$ will change.

$|M_{12}|$ is measured from

$$\Delta m \simeq 2|M_{12}|.$$

$\phi = \arg M_{12} - \arg(-\Gamma_{12})$ enters

$$\Delta\Gamma \simeq 2|\Gamma_{12}|\cos\phi, \qquad a_{\rm fs} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi.$$

Γ_{12} is a tree-level quantity and is difficult to change significantly in models of new physics. It is safe to assume $\Gamma_{12} = \Gamma_{12,\mathrm{SM}}$.
Then new physics can only enter $\Delta\Gamma$ via $\cos\phi$. Two effects:

- $\Delta\Gamma = \Delta\Gamma_{\mathrm{SM}} \cos\phi$.
- $|\, B_L \,\rangle$ and $|\, B_H \,\rangle$ are no more CP eigenstates.
  $\Rightarrow$ both $|\, B_L \,\rangle$ and $|\, B_H \,\rangle$ can decay into $(J/\psi\phi)_{L=0}$
  $\Rightarrow$ the lifetime measured in $\overset{(-)}{B}_s \to (J/\psi\phi)_{L=0}$ is no more $1/\Gamma_L$.

As a result the comparison of the width measured in this decay and Γ_{B_s} yields

$$\Delta\Gamma_{\mathrm{SM}} \cos^2\phi.$$

Grossman 1996, Dunietz, Fleischer, U.N. 2000

$\Rightarrow$ New physics contributions to M_{12} can only diminish $\Delta\Gamma$. Further the described measurements yield no information on the sign of $\Delta\Gamma$.

Better measurements to determine $\arg M_{12}$:

$$\sin 2 \left[\arg M_{12} - \arg(V_{cb}^2 V_{cs}^{*2}) \right]$$

is obtained from the time-dependent CP asymmetry in $B_s \to J/\psi\phi$.

An extremely sensitive quantity to measure $\arg M_{12}$ is $a_{\rm fs}$, which can be enhanced from $2 \cdot 10^{-5}$ to almost 10^{-2}.

A SO(10) GUT model

SUSY-GUT's unify quarks and leptons. E.g.

$$\overline{\mathbf{5}} = \begin{pmatrix} d_R^c \\ d_R^c \\ d_R^c \\ \ell_L \\ \nu_\ell \end{pmatrix} \qquad \text{in SU(5)}$$

Experiment: ν_μ–ν_τ mixing is large. If the large mixing angle comes from the rotation of a $\overline{\mathbf{5}}$ in flavour space, a large $\tilde{s}_R$–$\tilde{b}_R$ mixing is possible.

$\Rightarrow$ $B_s - \overline{B}_s$ mixing gets a large contribution from box diagrams with squarks and gluinos.

Chang, Masiero, Murayama (CMM), hep-ph/0205111:

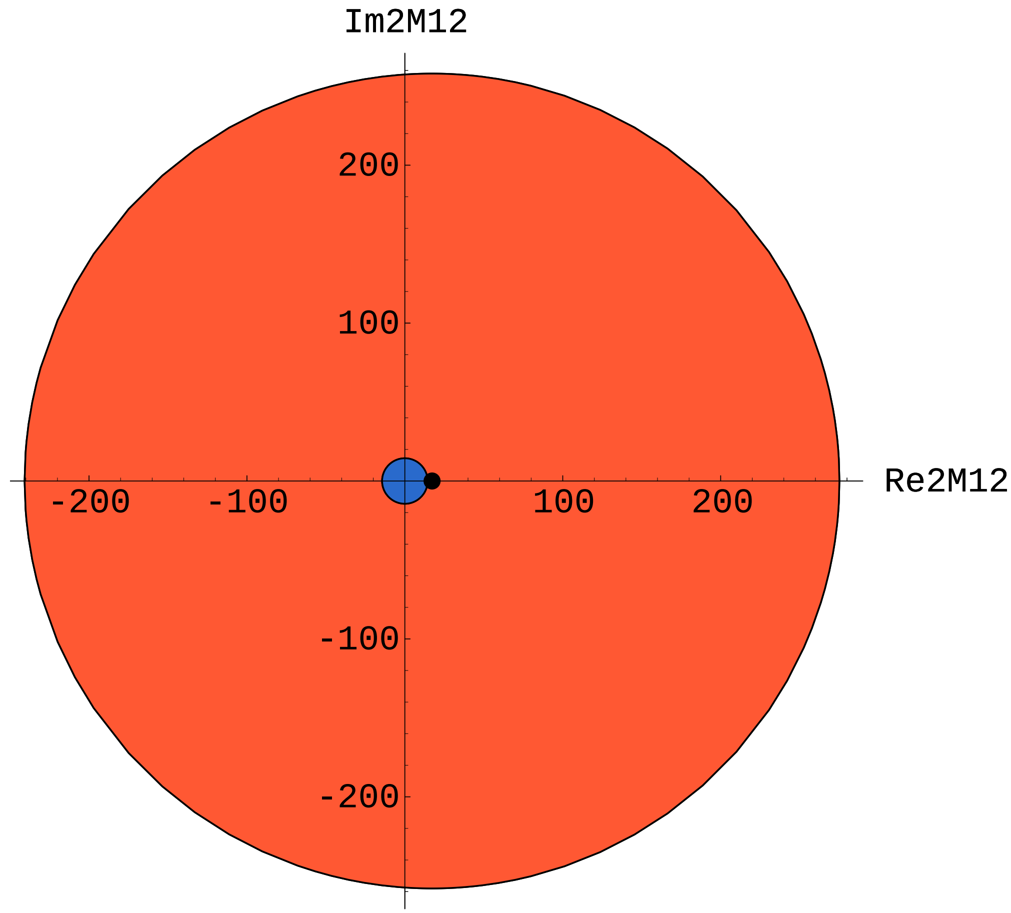
Model based on the breaking chain

$$\mathrm{SO}(10) \to \mathrm{SU}(5) \to \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$$

with SUSY-breaking terms universal near the Planck scale. Renormalisation group effects from the top Yukawa coupling destroy the universality. Large $\tilde{s}_R$–$\tilde{b}_R$ mixing is generated when the $\overline{\mathbf{5}}$ is rotated to the mass eigenstate basis.

With a renormalisation group analysis of CMM model the effect on $B_s-\overline{B}_s$ mixing has been quantified.

Jäger,UN,hep-ph/0312145



Here $2|M_{12}|$ is the $B_s - \overline{B}_s$ oscillation frequency and $\phi = \arg M_{12}$ is the new CP phase in $B_s - \overline{B}_s$ mixing.

Black: Standard Model prediction, Blue: experimentally excluded,
Red: allowed in the CMM model

The $B_s - \overline{B}_s$ oscillations are typically too large to be detected. But $\Delta\Gamma$ can be measured without resolving these oscillations and $\Delta\Gamma$ is diminished by a factor of $\cos\phi$.

4. Conclusions

$B_s - \overline{B}_s$ mixing is very sensitive to new physics affecting $\arg M_{12}$. The $B_s - \overline{B}_s$ oscillation frequency measures $\Delta m \simeq 2|M_{12}|$, while the phase of M_{12} is measured through $\Delta\Gamma$, $a_{\rm fs}$ or the CP asymmetry in $B_s \to J/\psi\phi$. The quantities $\Delta\Gamma$ and $a_{\rm fs}$ can be measured without resolving the rapid $B_s - \overline{B}_s$ oscillations. This opens a door to $\arg M_{12}$ in the case that Δm is too large to be measured. An interesting connection between the atmospheric neutrino mixing angle and $b \to s$ transitions exists in certain GUT models. $B_s - \overline{B}_s$ mixing is a superb testing ground for these models.